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SOME FINITE INTEGRALS INVOLVING THE I-FUNCTION OF TWO VARIABLES

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The aim of the present paper is to establish some finite integrals involving the products of I-function of two variables, defined by Goyal and Agrawal [5], The Jaccobi Polynomials and a general class of Polynomials.

INTRODUCTION: Goyal and Agrawal [5], defined the *I*–function of two variables, which is an extension of I-function of One variable defined given by Saxena V.P. [7], as :

$$I_{p\,,\,q\,:\,Q}^{m_1\,,\,n_1\,:\,Q}\left[\begin{matrix} z_1 \\ z_2 \end{matrix}\left|\begin{matrix} \left[\left(e_p : E_p , E_p\right)\right] : U \\ \left[\left(f_q : F_q , F'_q\right)\right] : U' \end{matrix}\right.\right]$$

$$= -\frac{1}{4\pi^2}\int_{L_1}\int_{L_2}\varphi_1(\xi)\,\varphi_2(\eta)\,\psi(\xi,\eta)\,z_1^{\xi}\,z_2^{\eta}\,d\xi\,d\eta \qquad \text{...(1.1)}$$

where

$$\Phi_1(\xi) = \frac{\prod_{j=1}^{m_2}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{n_2}\Gamma(1-a_j+\alpha_j\xi)}{\sum_{i=1}^{r}\left[\prod_{j=m_2+1}^{q_i^{(1)}}\Gamma(1-b_{ji}+\beta_{ji}\xi)\prod_{j=n_2+1}^{p_i^{(1)}}\Gamma(a_{ji}-\alpha_{ji}\xi)\right]} \qquad \text{...(1.2)}$$

REQUIRED RESULTS: We shall require the following definitions and known results in the paper.

The known result [1, p.284, Eq. (1) & (3)]

$$\int_{-1}^{1}(1-x)^{\alpha}(1+x)^{\lambda}P_k^{\alpha,\beta}(x)\,dx = \frac{2^{\alpha+\lambda+1}\,\Gamma(\lambda+1)\,\Gamma(\alpha+1)\,\Gamma(\lambda-\beta+1)}{\Gamma(\lambda-\beta-k+1)\,\Gamma(\alpha+\lambda+k+2)} \qquad \text{...(2.1)}$$

and

$$\int_{-1}^{1}(1-x)^{\alpha}(1+x)^{\lambda}P_k^{\alpha,\beta}(x)\,dx = \frac{2^{\sigma+\lambda+1}\,\Gamma(\sigma+1)\,\Gamma(\lambda+1)}{\Gamma(\sigma+\lambda+1)}\;{}_3F_2\left[\begin{matrix}-k\,;\,\alpha+\beta+k+1\,,\,\sigma+1 \\ \alpha+1\quad,\quad\alpha+\lambda+2\end{matrix};1\right] \qquad \text{...(2.2)}$$

For the Jacobi polynomials we have $P_k^{(\alpha\,,\,\beta)}(x)$ [6, p. 254, Eq. (1)], we have

$$P_k^{(\alpha\,:\,\beta)}(t+\rho)\,P_k^{(\alpha\,,\,\beta)}(t-\rho) = \frac{(-1)^k\,(1+\alpha)_k\,(1+\beta)_k}{(k\,!)^2}$$

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$$\sum_{R=0}^{k} \frac{(-k)_R (1+\alpha+\beta+k)_R}{(1+\alpha)_R (1+\beta)_R} P_R^{(\alpha,\beta)}(x)\, t^R \quad \dots (2.3)$$

$$\rho^k P_k^{(\alpha,\alpha)}\left(\frac{1-xt}{\rho}\right) = \frac{(1+\alpha)_k}{k!} \sum_{R=0}^{k} \frac{(-k)_R}{(1+\alpha)_R} P_R^{(\alpha,\alpha)}(x)\, t^R \quad (2.4)$$

$$\frac{1}{\rho}(1-t+\rho)^{-\alpha}(1+t+\rho)^{-\beta} = 2-\alpha-\beta \sum_{R=0}^{\infty} P_R^{(\alpha,\beta)}(x)\, t^R \quad (2.5)$$

In each of the formulae (2.3), (2.4) and (2.5) and throughout the paper $\rho = (1-2xt+t^2)^{-1/2}$. The general class of polynomials [6, p. 1, Eq. (1)]

$$S_n^m[x] = \sum_{l=0}^{[\frac{n}{m}]} \frac{(-n)_{ml}}{l!} A_{n,l}\, x^l, \; n = 0, 1, 2, \dots \quad \dots (2.6)$$

$$\Phi_2(\eta) = \frac{\prod_{j=1}^{m_3} \Gamma(d_j - \delta_j \eta) \prod_{j=1}^{n_3} \Gamma(1 - c_j + \gamma_j \eta)}{\sum_{i=1}^{r} \left[\prod_{j=m_3+1}^{q_i^{(2)}} \Gamma(1 - d_{ji} + \delta_{ji} \eta) \prod_{j=n_3+1}^{p_i^{(2)}} \Gamma(c_{ji} - \gamma_{ji} \eta) \right]} \quad \dots (1.3)$$

$$\psi(\xi,\eta) = \frac{\prod_{j=1}^{m_1} \Gamma(f_j - F_j\xi - F_j\eta) \prod_{j=1}^{n_1} \Gamma(1 - e_j + E_j\xi + E_j\eta)}{\sum_{j=m_1+1}^{q} \Gamma(1 - f_j + F_j\xi + F_j\eta) \prod_{j=n_1+1}^{p} \Gamma(e_j - E_j\xi - E_j\eta)} \quad \dots (1.4)$$

The double integral in (1.1) converges absolutely if

$$|arg\, z_1| < \frac{A\pi}{2}, \; |arg\, z_2| < \frac{B\pi}{2} \quad \dots (1.5)$$

where

$$A = \sum_{1}^{n_1} E_j - \sum_{n_1+1}^{p} E_j + \sum_{1}^{m_1} F_j - \sum_{m_1+1}^{q} F_j + \sum_{1}^{m_2} \beta_j - \sum_{m_2+1}^{q_i^{(1)}} \beta_{ji} + \sum_{n_2}^{n_2} \alpha_j - \sum_{n_2+1}^{p_i^{(1)}} \alpha_{ji} > 0 \quad \dots (1.6)$$

$$B = \sum_{1}^{n_1} E_j - \sum_{n_1+1}^{p} E_j + \sum_{1}^{m_1} F_j - \sum_{m_1+1}^{q} F_j + \sum_{1}^{m_3} \delta_j - \sum_{m_3+1}^{q_i^{(2)}} \delta_{ji} + \sum_{1}^{n_3} \gamma_j - \sum_{n_3+1}^{p_i^{(3)}} \gamma_{ji} > 0 \quad \dots (1.7)$$

and the notations

$$Q = m_2, n_2, m_3, n_3 \quad \dots (1.8)$$

$$Q = p_i^{(1)}, q_i^{(1)}, p_i^{(2)}, q_i^{(2)} : r \quad \dots (1.9)$$

$$U = [(a_j, \alpha_j)_{1,n_2}], [(a_{ji}, \alpha_{ji})_{n_2+1, p_i^{(1)}}]; [(c_j, \gamma_j)_{1,n_3}], [(c_{ji}, \gamma_{ji})_{n_3+1, p_i^{(2)}}] \quad \dots (1.10)$$

$$U = [(b_j, \beta_j)_{1,m_2}], [(b_{ji}, \beta_{ji})_{m_2+1, q_i^{(1)}}]; [(d_j, \delta_j)_{1,m_3}], [(d_{ji}, \delta_{ji})_{m_3+1, q_i^{(2)}}] \dots (1.11)$$

Throughout the paper, we use the notations Q, Q', U, U' as per equations (1.8) to (1.11) respectively.

where m is an arbitrary positive integer and the coefficients $A_{n,l}$ $(n, l \geq 0)$ are arbitrary constants, real or complex.

By suitably specializing the coefficients $A_{n,l}$, the polynomials $S_n^m[x]$ can be reduced to the well known classical orthogonal polynomials such as Jaccobi, Hermite, Legendre polynomials etc.

FINITE INTEGRALS: We shall establish the following finite integrals

$$\text{(I)} \qquad \int_{-1}^{1} (1-x)^{\alpha}(1+x)^{\lambda} P_k^{(\alpha,\beta)}(x) S_n^m[(1+x)^{\theta}]\, I\begin{bmatrix}(1+x)^{k_1} z_1\\(1+x)^{k_2} z_2\end{bmatrix} dx$$

$$= 2^{\alpha+\lambda+1}\Gamma(\alpha+k+1)\sum_{l=0}^{[\frac{n}{m}]} \frac{2^{\theta l}(-n)_{ml}}{l!} A_{n,l}\, I^{m_1, n_1+2:Q}_{p+2, q+2:Q}\left[\begin{matrix}2^{k_1} z_1\\2^{k_2} z_2\end{matrix}\left|\begin{matrix}X : U\\X : U\end{matrix}\right.\right] \qquad \text{...(3.1)}$$

where

$$X = (-\lambda-\theta l : k_1, k_2), (\beta-\lambda-\theta l : k_1, k_2), (e_p : E^p, E_p')$$

$$X' = (f_q ; F_q, F_q'), (\beta-\lambda-\theta l+k : k_1, k_2), (-1-\alpha-\lambda-\theta l-k : k_1, k_2)$$

provided

$$\theta, k_1, k_2 > 0\ ; \quad \mathrm{Re}(\alpha) > -1\ ;$$

$$\mathrm{Re}\left[\lambda+\theta l+k_1 \min_{1\leq j\leq m_2}\left(\frac{b_j}{\beta_j}\right)+k_2 \min_{1\leq j\leq m_3}\left(\frac{d_j}{\delta_j}\right)\right] > -1\ ;$$

$$|\arg z_1| < \frac{A\pi}{2}, \ |\arg z_2| < \frac{B\pi}{2}$$

where A & $B > 0$, are given as per equations (1.6) & (1.7) and m is an arbitrary positive integer and the coefficients $A_{n,l}$ $(n, l \geq 0)$ are arbitrary constants, real or complex.

$$\text{(II)} \quad \int_{-1}^{1} (1-x)^{\sigma}(1+x)^{\lambda} P_k^{(\alpha,\beta)}(x) S_n^m\left[(1-x)^{\theta_1}(1+x)^{\theta_2}\right] I\begin{bmatrix}(1-x)^{h_1}(1+x)^{k_1} z_1\\(1-x)^{h_2}(1+x)^{k_2} z_2\end{bmatrix} dx$$

$$= 2^{\sigma+\lambda+1}\sum_{l=0}^{[\frac{n}{m}]}\sum_{\varepsilon=0}^{\infty} \frac{2^{(\theta_1+\theta_2)l}(-n)_{ml}}{l!} A_{n,l} \frac{(-k)_{\varepsilon}(\alpha+\beta+k+1)_{\varepsilon}}{\varepsilon!(\alpha+1)_{\varepsilon}} *$$

$$I^{m_1, n_1+2:Q}_{p+2, q+1:Q}\left[\begin{matrix}2^{h_1+k_1} z_1\\2^{h_2+k_2} z_2\end{matrix}\left|\begin{matrix}X : U\\X' : U'\end{matrix}\right.\right] \qquad \text{...(3.2)}$$

where

$$X = (-\sigma-\theta_1 l-\varepsilon : h_1, h_2), (-\lambda-\theta_2 : k_1, k_2), (e_p : E_p, E_p')$$

$$X' = (f_q : F_q, F_q'), (-1-\sigma-\lambda-\varepsilon-(\theta_1+\theta_2) l : h_1+k_1, h_2+k_2)$$

Provided $h_1, h_2, k_1, k_2, \theta_1, \theta_2 > 0$;

$$\text{Re}\left[\sigma+\theta_1 l+h_1 \underset{1\le j\le m_2}{Min}\left(\frac{b_j}{\beta_j}\right)+h_2 \underset{1\le j\le m_3}{Min}\left(\frac{d_j}{\delta_j}\right)\right] > -1;$$

$$\text{Re}\left[\lambda+\theta_2 l+h_1 \underset{1\le j\le m_2}{Min}\left(\frac{b_j}{\beta_j}\right)+k_2 \underset{1\le j\le m_3}{Min}\left(\frac{d_j}{\delta_j}\right)\right] > -1;$$

$$\left|arg\, z_1\right| < \frac{A\pi}{2},\ \left|arg\, z_2\right| < \frac{B\pi}{2}$$

where A & $B > 0$ are given as per equations (1.6) & 91.7) and m is an arbitrary positive integer and the coefficients $A_{n,l}$ $(n, l \ge 0)$ are arbitrary constants, real or complex.

(III)

$$\int_{-1}^{1}(1-x)^{\alpha}(1+x)^{\lambda}\ P_k^{(\alpha,\beta)}\ (t+\rho)\ P_k^{(\alpha,\beta)}\ (t-\rho)\ S_n^m\ [\ (1+x)^{0}\ I\left[\begin{matrix}(1+x)^{k_1} z_1\\(1+x)^{k_2} z_2\end{matrix}\right] dx$$

$$=\frac{2^{\alpha+\lambda+1}(-1)^k\,\Gamma(1+\alpha+k)\,\Gamma(1+\beta+k)}{(k\,!)^2}\sum_{l=0}^{[\frac{n}{m}]}\sum_{\varepsilon=0}^{k}\frac{2^{\theta l}(-n)_{ml}}{l\,!}A_{n,l}\ *$$

$$\frac{(-k)_{\varepsilon}(1+\alpha+\beta+k)_{\varepsilon}}{\Gamma(1+\beta+\varepsilon)}\, t^{\varepsilon}\ I^{m_1,n_1+2:Q}_{p+2,q+2:Q'}\left[\begin{matrix}2^{k_1}z_1\\2^{k_2}z_2\end{matrix}\middle|\begin{matrix}X:U\\X':U'\end{matrix}\right] \tag{3.3}$$

where

$$X=(-\lambda-\theta l:k_1,k_2),(\beta-\lambda-\theta l:k_1,k_2),(e^{P}:E_p,E_p')$$

$$X'=(f_q:F_q,F_q'),(\beta-\lambda-\theta l+\varepsilon:k_1,k_2),(-1-\alpha-\lambda-\theta l-\varepsilon:k_1,k_2)$$

and all conditions are same as given in (3.1)

(IV) $$\int_{-1}^{1}(1-x)^{\alpha}(1+x)^{\lambda}\ \rho^k\,P_k^{(\alpha,\alpha)}\left(\frac{1-xt}{\rho}\right)S_n^m[\,(1+x)^{\theta}\,]\ I\left[\begin{matrix}(1+x)^{k_2} z_1\\(1+x)^{k_2} z_2\end{matrix}\right] dx$$

$$=\frac{2^{\alpha+\lambda+1}\Gamma(1+\alpha+k)}{k\,!}\sum_{l=0}^{[\frac{n}{m}]}\sum_{\varepsilon=0}^{k}\frac{2^{\theta l}(-n)_{ml}}{l\,!}A_{n,l}\ *$$

$$(-k)_{\varepsilon}\, t^{\varepsilon}\ I^{m_1,n_1+2\ :\ Q}_{p+2,q+2:Q}\left[\begin{matrix}2^{k_1}z_1\\2^{k_2}z_2\end{matrix}\middle|\begin{matrix}X:U\\X':U'\end{matrix}\right] \quad \text{...(3.4)}$$

where

$$X=(-\lambda-\theta l:k_1,k_2),(\alpha-\lambda-\theta l:k_1,k_2),(e_p:E_p,E_p')$$

$$X'=(f_q:F_q,F_q'),(\alpha-\lambda-\theta l+\varepsilon:k_1,k_2),(-1^{\theta}\ \alpha-\lambda-\theta l-\varepsilon:k_1,k_2)$$

and all conditions are same as given in (3.1).

(V) $$\int_{-1}^{1} (1-x)^{\alpha}(1+x)^{\lambda}\,\frac{1}{\rho}\,(1-t+\rho)^{-\alpha}(1-t+\rho)^{-\beta}\,S_n^m\,[\,(1+x)^{\theta}\,]\;I\left[\begin{matrix}(1+x)^{k_1}z_1\\ 1+x)^{k_2}z_2\end{matrix}\right]dx$$

$$=2^{\lambda-\beta+1}\sum_{\varepsilon=0}^{\infty}\sum_{l=0}^{[\frac{n}{m}]}\frac{2^{\theta l}(-n)_{ml}}{l\,!}\,A_{n,l}\,\Gamma(t+\varepsilon+1\,t^{\varepsilon}\;I^{\,m_1,\,n_1+2\,:\,Q}_{\,p+2,\,q+2\,:\,Q}\left[\begin{matrix}2k^{1}z_1\\ 2^{k_2}z_2\end{matrix}\left|\begin{matrix}X\,:\,U\\ X\,:\,U\end{matrix}\right.\right] \quad \ldots(3.5)$$

where

$$X=(-\lambda-\theta l:k_1,k_2),(\beta-\lambda-\theta l:k_1,k_2),(e_p:E_p,E_p')$$

$$X'=(f_q:F_q,F_q'),(\beta-\lambda-\theta l+\varepsilon:k_1,k_2),(-1-\alpha-\lambda-\theta l-\varepsilon:k_1,k_2)$$

and all conditions are same as given in (3.1).

(VI) $$\int_{-1}^{1}(1-x)^{\sigma}(1+x)^{\lambda}\,P_k^{(\alpha,\beta)}(t+\rho)\,P_k^{(\alpha,\beta)}(t-\rho)\,S_n^m\Big[\,(1-x)^{\theta_1}(1+x)^{\theta_2}\Big]$$

$$I\left[\begin{matrix}(1-x)^{h_1}(1+x)^{k_1}z_1\\ (1+x)^{h_2}(1+x)^{k_2}z_2\end{matrix}\right]dx$$

$$=\frac{2^{\sigma+\lambda+1}(-1)^k\,\Gamma(1+\alpha+k)\,\Gamma(1+\beta+k)}{(k\,!)^2}\sum_{R=0}^{k}\sum_{l=0}^{\left[\frac{n}{m}\right]}\sum_{\varepsilon=0}^{\infty}\frac{(-k)_R(1+\alpha+\beta+k)_R}{\Gamma(1+\alpha+R)\Gamma(1+\beta+R)}$$

$$\frac{2^{(\theta_1+\theta_2)l}(-n)_{ml}}{l\,!}\,A_{n,l}\,\frac{(-R)_{\varepsilon}(\alpha+\beta+R+1)_{\varepsilon}}{\varepsilon\,!\,(1+\alpha)_{\varepsilon}}\,t^{R}\;I^{\,m_1,\,n_1+2\,:\,Q}_{\,p+2,\,q+1\,:\,Q'}\left[\begin{matrix}2^{h_1+k_1}z_1\\ 2^{h_2+k_2}z_2\end{matrix}\left|\begin{matrix}X\,:\,U\\ X'\,:\,U'\end{matrix}\right.\right] \quad \ldots(3.6)$$

where

$$X=(-\sigma-\theta_1 l-\varepsilon:h_1,h_2),(-\lambda-\theta_2 l:k_1,k_2),(e_p:E_p,E_p')$$

$$X'=(f_q:F_q:F_q'),(-1-\sigma-\lambda-\varepsilon-(\theta_1+\theta_2)\,l:h_1+k_1,h_2+k_2)$$

and all conditions are same as given in (3.2).

(VII)

$$\int_{-1}^{1}(1-x)^{\sigma}(1+x)^{\lambda}\,\rho^{k}\,P_k^{(\alpha,\alpha)}\left(\frac{1-xt}{\rho}\right)S_n^m\Big[\,(1-x)^{\theta_1}(1+x)^{\theta_2}\Big]\,I\left[\begin{matrix}(1-x)^{h_1}(1+x)^{k_1}z_1\\ (1+x)^{h_2}(1+x)^{k_2}z_2\end{matrix}\right]dx$$

$$=\frac{2^{\sigma+\lambda+1}\,\Gamma(1+\alpha+k)}{k\,!}\sum_{R=0}^{k}\sum_{t=0}^{\left[\frac{n}{m}\right]}\sum_{\varepsilon=0}^{\infty}\frac{2^{(\theta_1+\theta_2)l}\,(-n)_{ml}}{l\,!}\,A_{n,l}\,\frac{(-k)\,R\,t^{R}}{\Gamma(1+\theta+R)}$$

$$\frac{(-R)_\varepsilon (2\alpha + R + 1)_\varepsilon}{\varepsilon!\,(1+\alpha)_\varepsilon}\; I^{m_1, n_1 + 2 : Q}_{p+2, q+1 : Q'}\left[\begin{matrix} 2^{h_1+k_1} z_1 \\ 2^{h_2+k_2} z_2 \end{matrix}\middle| \begin{matrix} X : U \\ X' : U' \end{matrix}\right] \qquad \ldots (3.7)$$

where

$$X = (-\sigma - \theta_1 l - \varepsilon : h_1, h_2), (-\lambda - \theta_2 l : k_1, k_2), (e_p : E_p, E_p')$$

$$X' = (f_q : F_q, F_q'), (-1 - \sigma - \lambda - \varepsilon(\theta_1 + \theta_2) l : h_1 + k_1, h_2 + k_2)$$

and all conditions are same as given in (3.2).

(VIII)

$$\int_{-1}^{1} (1-x)^\sigma (1+x)^\lambda \frac{1}{\rho} (1 - t + \rho)^{-\alpha} (1 + t + \rho)^{-\beta} S_n^m \left[(1-x)^{\theta_1} (1+)^{\theta_2} \right]^{*}$$

$$I\left[\begin{matrix} (1-x)^{h_1} (1+x)^{k_1} z_1 \\ (1+x)^{h_2} (1+x)^{k_2} z_2 \end{matrix}\right] dx$$

$$= 2^{-\alpha - \beta + \sigma + \lambda + 1} \sum_{R=0}^{\infty} \sum_{l=0}^{[n/m]} \sum_{\varepsilon=0}^{R} \frac{2^{(\theta_1 + \theta_2) l} (-n)_{ml}}{l!} A_{n,l}$$

$$\frac{(-R)_\varepsilon (\alpha + \beta + R + 1)_\varepsilon}{\varepsilon!\,(1+\alpha)_\varepsilon}\, t^R\, I^{m_1, n_1 + 2 : Q}_{p+2, q+1 : Q'}\left[\begin{matrix} 2^{h_1+k_1} z_1 \\ 2^{h_2+k_2} z_2 \end{matrix}\middle| \begin{matrix} X : U \\ X' : U' \end{matrix}\right] \qquad \ldots (3.8)$$

where

$$X = (-\sigma - \theta_1 l - \varepsilon : h_1, h_2), (-\lambda - \theta_2 l : k_1, k_2), (e_p : E_p, E_p')$$

$$X' = (f_q : F_q : F_q'), (-1 - \sigma - \lambda - \varepsilon - (\theta_1 + \theta_2) l : h_1 + k_1, h_2 + k_2)$$

and all conditions are same as given in (3.2).

PROOF: The integrals (3.1) & (3.2) may be evaluated by making use of the known results (2.1) & (2.2), a general class of Polynomials (2.6) and the definition of *I*-function of two variables given in (1.1).

The proofs of the formulae (3.3) to (3.8) can be derived by the help of the results (3.1) & (2.3), (3.1) and (2.4), (3.1) & (2.5), (3.2) & (2.3), (3.2) & (2.4), (3.2) & (2.5) respectively.

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